Maximum of cooling force

(December 16, 2003)

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Standard statement which can be found in any paper on electron cooling theory:

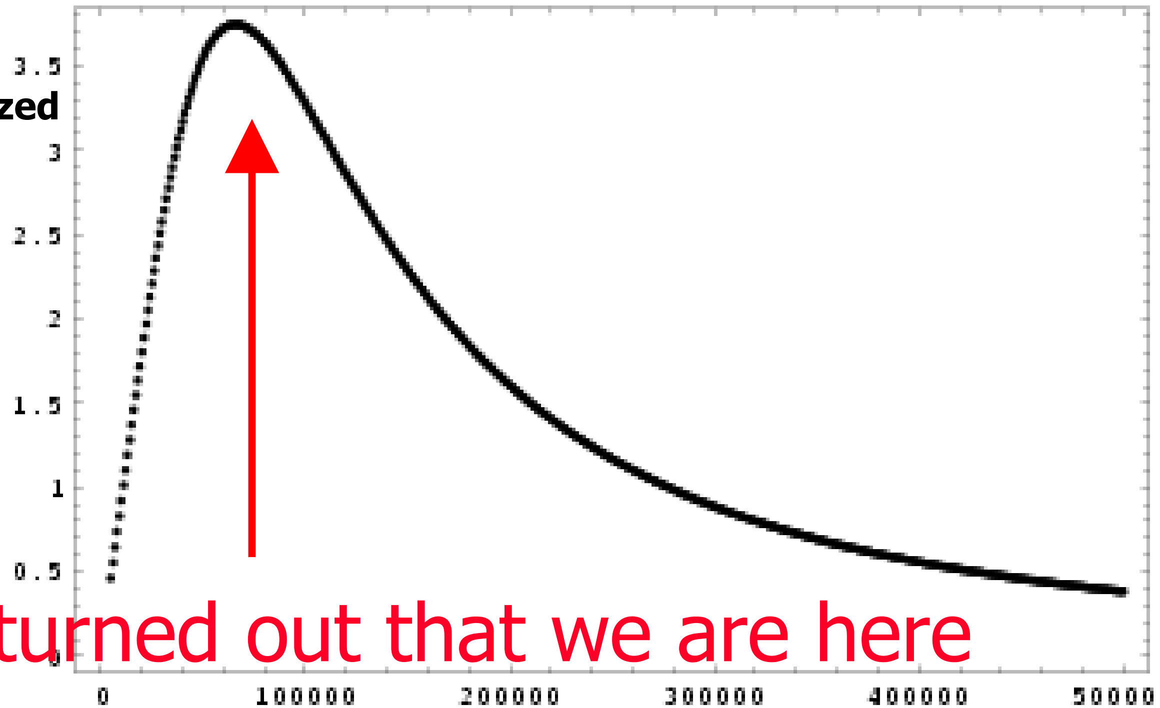
"Maximum of magnetized cooling force happens at longitudinal electron velocity"

Cool


we wanted to be here
3.5
3
2.5
2
1.5
1
0.5
0
0
100000
200000
300000
400000
500000

Electron longitudinal velocity in simulation was chosen to be very low.

Calculated F_{cool} based on VP formula for "scaled-1" parameters used in Vorpal simulations



This resulted in studies of VP formula:

1) Role of cooling Log

2) Role of Veff

3) Where to expect maximum of the force

VP formula

1) One can go back to collision integral or

2) Obtain scaling from a simple formula

V. Parkhomchuck (VP) - empiric

$$\mathbf{F} = -\frac{1}{\pi}\omega_{pe}^{2}\frac{(Ze)^{2}}{4\pi\varepsilon_{0}}\ln\left(\frac{\rho_{\max}+\rho_{\min}+r_{L}}{\rho_{\min}+r_{L}}\right)\frac{\mathbf{V}_{ion}}{\left(V_{ion}^{2}+V_{eff}^{2}\right)^{3/2}}$$

$$\ln\left(\frac{\rho_{max}+\rho_{min}}{\rho_{min}}\right) \approx \frac{V^3}{c^3}\frac{1}{\sqrt{4\pi r_e^3}}$$

$$\bar{F} = 2e^2 n_e^{2/3} \frac{\bar{V}}{c\sqrt{\pi n_e^{1/3} r_e}}$$

$$V = c\sqrt{\pi n_e^{1/3} r_e}$$

$$T_{||} = m\Delta_{e||}^2 \approx \frac{T_c^2}{2E_0} + 2e^2 n_e^{1/3}$$

Typical – “plasma case”

Maximum

1. Very low longitudinal velocity:

1) Plasma case

$$V_{\max} = c\left(\omega_p Z r_e / c\right)^{1/3}$$

Turns out to be Electrostatic long. velocity

2) τ case:

$$V_{\max} = c\left(Z\beta\gamma r_e / L\right)^{1/3}$$

Nothing to do with electrost. Long. velocity

2) High longitudinal velocity:

In RF acceleration Ve_long is in fact much bigger than Δe_electrost.

In such case Ve_long is determined by achievable energy spread and is an order of magnitude higher than Δe_electrost.

Here, Veff plays a crucial role in indicating maximum of the force:

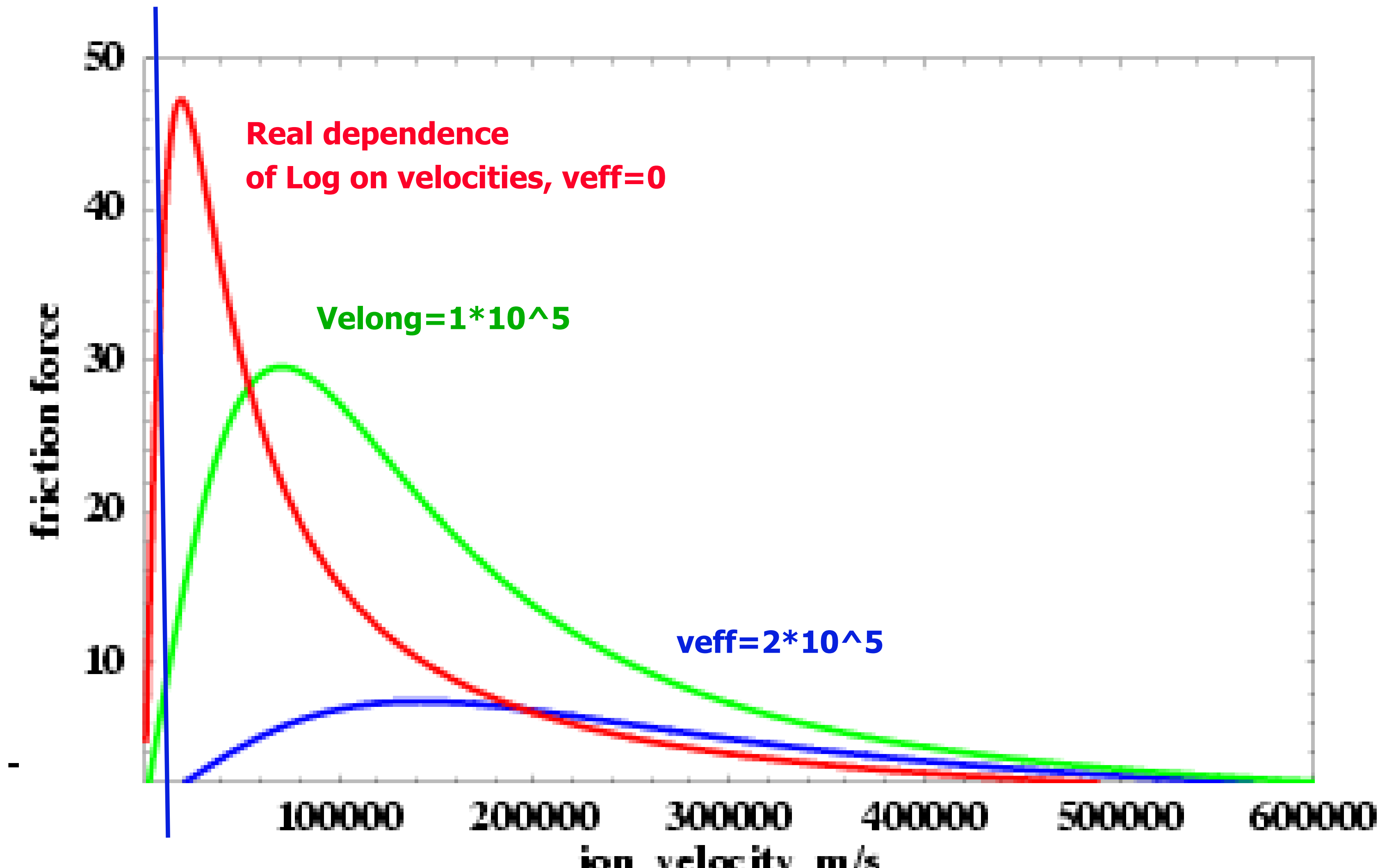
In principle: Veff^2=Vsc^2+Verr^2+Ve_long^2 – again 3 different possibilities depending on the machine

- For RHIC – Vsc -> 0

Solenoidal error – is now slightly above Ve_long.

1. If solenoid error is further improved than maximum moves to Ve_long, determined by energy spread in the linac
2. If Ve_long (energy spread is improved) than maximum moves even further towards low velocity

Dependence on Veffective



- Two numerical experiments were performed with Vorpal code:

1. Ultra low electron longitudinal velocity: Δe1
2. Relatively high electron longitudinal velocity which determined

Veffective: Δe2

Results with Vorpal code